Non-Gaussian features from the inverse volume corrections in loop quantum cosmology

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Abstract

In this paper we study the non-Gaussian features of the primordial fluctuations in loop quantum cosmology with the inverse volume corrections. The detailed analysis is performed in the single field slow-roll inflationary models. However, our results reflect the universal characteristics of bispectrum in loop quantum cosmology. The main corrections to the scalar bispectrum come from two aspects: one is the modifications to the standard Bunch-Davies vacuum, the other is the corrections to the background dependent variables, such as slow-roll parameters. Our calculations show that the loop quantum corrections make $f_{\rm NL}$ of the inflationary models increase 0.1%. Moreover, we find that two new shapes of non-Gaussian signal arise, which we name $\mathcal{F}_1$ and $\mathcal{F}_2$. The former gives a unique loop quantum feature which is less correlated with the local, equilateral and single types, while the latter is highly correlated with the local one.

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I. INTRODUCTION

As a non-perturbative and background-independent theory, Loop Quantum Gravity (LQG) [1–3] has achieved great successes in past years: derivations of the quantized area and volume operators [4–7], calculations of black holes entropy [8] and Loop Quantum Cosmology(LQC) [9], *etc.* And the nonperturbative quantization procedure of LQG is also valid for a more general class of four-dimensional metric theories of gravity [10–12]. As an example of LQG, LQC gives a quantization scheme of LQG for a symmetry-reduced model in the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker universe. The discrete spacetime geometry in LQC scenarios predicts a non-singular bouncing universe in some simplified models, which satisfies most of the astronomical and cosmological observational constraints. Although the quantum correction effects are being diluted with the expansion of our universe, it remains present in a weaker form, especially on/near the super-horizon scales.

Recently, the gauge invariant cosmological perturbation theory has been systematically constructed in [13–15] for inverse volume corrections and in [16, 17] for holonomy corrections. Some relevant applications have been considered in [18, 19, 21, 22]. The inverse volume corrections and the holonomy corrections are two main quantum corrections in LQC. The inverse volume corrections come from the quantization of the inverse of the volume operator in LQG. The inverse volumes exist in the Hamiltonian constraint of gravity and the usual matter Hamiltonian, especially in the kinematic terms. Since the volume can be taken the value zero, there does not exist well defined inverses of the volume operator. Fortunately, with the Thiemann trick [23], we can construct well defined inverse volume operators, which bring the quantum corrections. While the holonomy corrections arise from the loop quantization based on the holonomies instead of the direct connection. The holonomy corrections become important when the energy scale of our Universe approaches the Planck one. Both the modifications to the scalar and tensor primordial power spectra from the inverse volume corrections are carefully investigated by the authors of [18]. Their results show that the inverse volume corrections could give rise to the enhancement of the power spectra on the large scales, i.e., a red-tilt one. However, some other mechanisms such as the non-commutative geometry or string theory [24, 26, 27], could also lead to similar features. Therefore to seek for signature of loop quantum cosmology, the study for the non-Gaussianity features in loop

quantum cosmology is necessary.

Because the primordial non-Gaussianities are quite helpful to distinguish inflationary models, so far a lot of papers have been devoted to studying the non-Gaussianities in different inflation models, see the relevant references in the nice reviews [28, 29]. Inspired by the studies of [18], in this paper we mainly consider the non-Gaussianities from the inverse volume corrections in LQC. The reason for consider inverse volume corrections only is as following. We denote $\delta_{\rm inv}$ as the correction term coming from the inverse volume operator and $\delta_{\rm hol}$ as the correction term from the holonomy corrections. We can estimate the inverse volume correction as[21]

$$\delta_{\rm inv} \sim \left(\frac{8\pi}{3}\frac{\rho}{\rho_{\rm Pl}}\delta_{\rm hol}^{-1}\right)^2 , \tag{1}$$

where the Planck density $\rho_{\rm Pl}$ is assumed as the quantum gravity scale. From the above expression, we can see that the inverse volume corrections behave very differently from what is normally expected for quantum gravity. For low densities, the holonomy corrections is small, but the inverse volume one may still be large because they are magnified by the inverse of $\delta_{\rm hol}$. For an example, the small holonomy corrections of size $\delta_{\rm hol} < 10^{-6}$ then requires the inverse volume corrections larger than $\delta_{\rm inv} > 10^{-6}$ even at scale $\rho \approx 10^{-9}\rho_{\rm Pl}$. These novel features make the investigations on the inverse volume corrections more interesting than the former at sub-Planckian inflationary scales. So we only consider the inverse volume correction in this work.

Explicitly, in the perturbation theory in LQC, the inverse volume operator can be captured by a correction function such as $\bar{\alpha} \simeq 1 + \alpha_0\delta_{\rm inv} \simeq 1 + \alpha_0(a_{\rm inv}/a)^\sigma$, where a is the scale factor of the FLRW universe and $a_{\rm inv}$ is introduced to describe the characteristic scale of the inverse volume correction, which is not the Planck one in general. When $a_{\rm inv}/a \ll 1$, we can ignore the correction term. However, if $a_{inv}/a \lesssim 1$ during inflation, one cannot neglect inverse volume corrections. In this case, the correction approximates $\alpha_0\delta_{\rm inv}(k) \approx \delta(k_0)(k_0/k)^\sigma$, where k and k_0 are, respectively, the considered perturbation wave number and some characteristic number involved in the inverse volume correction. In addition, many works about LQC imply that $\sigma \in [0, 6]$ [19]. From the form of the inverse volume correction, we can see that a small sigma corresponds to a small the inverse volume correct, vice versa. Therefore, as an example, following [19], we take $\sigma = 2$ in this paper. For other σ values the behavior will be similar. Furthermore, in terms of spherical multiples the wave number could be

expressed as $k \approx 10^{-4} hl$, with h representing for the reduced Hubble parameter $h = 0.7$ and l for the spherical multiples. In the typical linear regime of Cosmic Microwave Background (CMB), the multiples l range in $2 < l < 1000$ or more. Given all the mentioned observations, we expect some new features in the non-Gaussianities will arise. The purpose of this work is to investigate the characterized sizes and shapes of bispectra in LQC scenarios.

Note that the bispectra for the single field slow-roll inflationary model have been calculated in the papers [30–34]. For simplicity and comparison, we study the simplest single field slow-roll inflationary model in LQC. Our results show that the quantum corrections mainly come from the third order interaction Hamiltonian, the corrected vacuum state and the corrections to the slow-roll parameters. Our paper is organized as follows. In Sec. II, the canonical formulism and the slow-roll inflationary model in LQC scenarios are briefly reviewed. In Sec. III, we study the power spectrum in LQC and recover the previous results. The effect of the inverse volume corrections on the non-Gaussianity in LQC is investigated in Sec. IV. The detailed analysis for the sizes, shapes and shape correlations is presented in Sec. IVand Sec. V, respectively. Throughout this paper we set $8\pi\gamma G = 1$ and Einstein's summing convention is always adopted.

II. REVIEW OF LOOP QUANTUM COSMOLOGY

The framework of LQG/LQC will be briefly presented in this section. Firstly, we discuss the canonical formalism in LQG, and then introduce the dynamics of slow-roll inflationary models in LQC scenarios.

A. The canonical formalism in loop quantum gravity

In the framework of LQG [1, 3], the spatial metric as a canonical field is replaced by the densitized triad E^a_i, defined as

$$E^a_i := |\det(e^j_b)| e^a_i, \tag{2}$$

where e^a_i is the inverse of the cotriad e^i_a related to the spatial metric by $q_{ab} = e^i_a e^i_b$. The canonically conjugate variable to the densitized triad is the Ashtekar-Barbero connection $A^i_a := \Gamma^i_a + \gamma K^i_a$, where K^i_a is the extrinsic curvature and $\gamma \approx 0.274$ is the Barbero-Immirzi

parameter[37, 38]. The densitized triad E^a_i and the Ashtekar-Barbero connection A^i_a satisfy the following commutator relation

$$\{A^i_a(x), E^b_j(y)\} = \delta^b_a \delta^i_j \delta^3(x, y). \tag{3}$$

The spin connection Γ^i_a is defined such that it leaves the triad covariantly constant and has the explicit form

$$\Gamma^i_a = -\epsilon^{ijk} e^b_j (\partial_{[a} e^l_{b]} \delta_{lk} + \frac{1}{2} e^c_k e^l_a \partial_{[c} e^m_{b]} \delta_{lm}). \tag{4}$$

In the new Ashtekar variables, the Einstein-Hilbert action can be expressed in the canonical form

$$S_{EH} = \int dt \Big[\int_\Sigma d^3x \dot{K}^i_a E^a_i - \mathscr{D}_{\rm grav}[N^a] - \mathscr{H}_{\rm grav}[N] - \mathscr{G}_{\rm grav}[\Lambda^i] \Big] \tag{5}$$

where the Diffeomorphism constraint is

$$\mathscr{D}_{\rm grav}[N^a] = \frac{1}{\gamma} \int_\Sigma d^3x N^a \Big[(\partial_a A^j_b - \partial_b A^j_a) E^b_j - A^j_a \partial_b E^b_j \Big]. \tag{6}$$

And correspondingly the Hamiltonian constraint can be expressed as

$$\mathscr{H}_{\rm grav}[N] = \frac{1}{2} \int_\Sigma d^3x N \epsilon^{jk}_i \frac{E^c_j E^d_k}{\sqrt{|\det E|}} \Big[2\partial_c \Gamma^i_d + \epsilon^i_{mn} (\Gamma^m_c \Gamma^n_d - K^m_c K^n_d) \Big]. \tag{7}$$

The Gaussian constraint is

$$\mathscr{G}_{\rm grav}[\Lambda^i] = \int_\Sigma d^3x \Lambda^i (\partial_a E^a_i + \epsilon^k_{ij} \Gamma^j_a E^a_k + \epsilon^k_{ij} K^j_a E^a_k), \tag{8}$$

which can be solved through standard procedure [14]. Thus, solutions for the scalar mode perturbations are completely determined by the Hamiltonian constraint and the Diffeomorphism constraint.

B. Slow-roll inflationary models

In this subsection, we shortly review the inflationary dynamics of the Friedmann-Lemaître-Robertson-Walker Universe in LQC scenarios. The modified Friedmann equation, Raychaudhuri equation and Klein-Gordon equation are respectively [14]

$$\mathcal{H}^2 = \frac{1}{3} \bar{\alpha} \left(\frac{\bar{\varphi}'^2}{2\bar{\nu}} + \bar{p} V(\bar{\varphi}) \right), \tag{9}$$

$$\mathcal{H}' = \mathcal{H}^2 \left(1 + \frac{\bar{\alpha}_{,\bar{\rm p}} \bar{p}}{\bar{\alpha}} \right) - \frac{1}{2} \frac{\bar{\alpha}}{\bar{\nu}} \bar{\varphi}'^2 \left(1 - \frac{\bar{\nu}_{,\bar{\rm p}} \bar{p}}{3\bar{\nu}} \right), \tag{10}$$

$$\bar{\varphi}'' + 2\mathcal{H}^2 \bar{\varphi}' \left(1 - \frac{\bar{\nu}_{,\bar{\rm p}} \bar{p}}{\bar{\nu}} \right) + \bar{\nu} \bar{p} V_{,\varphi}(\bar{\varphi}) = 0, \tag{11}$$

where $\mathcal{H} = \frac{\bar{p}'}{2\bar{p}}$, $\bar{\nu}_{,\bar{\mathrm{p}}} \equiv d\bar{\nu}/d\bar{p}$, $\bar{p} \equiv a^2$ and a prime represents the derivative with respect to the conformal time. $\bar{\alpha}$ and $\bar{\nu}$ are the correction functions for the inverse volumes and they read $\bar{\alpha} \approx 1 + \alpha_0 \delta_{\mathrm{inv}}$, $\bar{\nu} \approx 1 + \nu_0 \delta_{\mathrm{inv}}$.

Following [19, 21], the slow-roll parameter ϵ can be straightforwardly calculated as

$$\begin{aligned} \epsilon &= 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} , \\ &= \epsilon_0 (1 + \gamma_\epsilon \delta_{\mathrm{inv}}) , \end{aligned} \tag{12}$$

where ϵ_0 denotes for the usual slow-roll parameter. And the explicit form of coupling constant γ_ϵ reads

$$\gamma_\epsilon = -\delta_0 \left[\frac{\sigma \alpha_0}{2\epsilon_0} + \alpha_0 + \nu_0 \left(\frac{\sigma}{6} - 1 \right) \right] . \tag{13}$$

Typically we set $\alpha_0 = 0.06$, $\nu_0 = 0.17$ and $\epsilon_0 = 0.01$ in this paper [19]. δ_0 is determined by quantum correction and the analysis after Eq.(43) implies that it takes $\mathcal{O}(10^{-3})$. As mentioned in Introduction we set $\sigma \in [0, 6]$ in this work. From above expression we can easily estimate that γ_ϵ is of the order $\mathcal{O}(10^{-3})$. Formally we can also express the another slow-roll parameter η as

$$\begin{aligned} \eta &= 1 - \frac{\varphi''}{\mathcal{H}\varphi'} , \\ &= \eta_0 (1 + \gamma_\eta \delta_{\mathrm{inv}}) , \end{aligned} \tag{14}$$

where η_0 denotes for the usual slow-roll parameter as ϵ_0.

As in the usual situation of the single field inflation model, we can assume that the two slow-roll parameters, ϵ_0 and η_0, take roughly the same order as 10^{-2}. Thus the typical values of coupling constant γ_η and γ_ϵ are of the order $\mathcal{O}(10^{-3})$. The terms proportional to the δ_{inv} represent for the inverse volume corrections. Here, we should emphasize that the subscript "inv" is introduced to avoid confusion with perturbations, such as $\delta\varphi$.

III. POWER SPECTRUM

In this section, we will firstly review the formalism of scalar perturbations in LQC; then, derive the second order Hamiltonian; and finally calculate the primordial power spectrum in the spatially flat gauge.

A. Formalism on the scalar modes

Consider the scalar perturbations only, the general form of a perturbed metric around the isotropic FRW background is

$$ds^2 = a^2(\tau)\Big\{ -(1+2\phi)d\tau^2 + 2\partial_a B d\tau dx^a + \Big[(1-2\psi)\delta_{ab} + 2\partial_a\partial_b E\Big]dx^a dx^b\Big\}, \tag{15}$$

where the scalar factor a is a function of the conformal time τ, and (ϕ, ψ, E, B) are the four scalar metric perturbations. In the perturbation theory, the triad can be described by

$$E_i^a = \bar{E}_i^a + \delta E_i^a , \tag{16}$$

where

$$\bar{E}_i^a = \bar{p}\delta_i^a, \qquad \delta E_i^a = -2\bar{p}\psi\delta_i^a + \bar{p}(\delta_i^a\Delta - \partial^a\partial_i)E. \tag{17}$$

The perturbed triad is described by the spatial part of the perturbed metric ψ and E. Here Δ is the laplace operator in the flat space. Similarly, the perturbed lapse function and shift vector can be described by the other two scalar metric perturbation ϕ and B respectively,

$$\delta N = \bar{N}\phi, \qquad \delta N^a = \partial^a B. \tag{18}$$

The extrinsic curvature can be perturbed as

$$K_a^i = \bar{K}_a^i + \delta K_a^i = \bar{k}\delta_a^i + \delta K_a^i. \tag{19}$$

For a general triad (16), the linearized spin connection becomes

$$\delta\Gamma_a^i = \frac{1}{2\bar{p}}\epsilon_a^{ij}\partial_b\delta E_j^b. \tag{20}$$

As described above, the symplectic structure also splits into two parts, one for the background variables and the other for the perturbations,

$$\{\bar{k}, \bar{p}\} = \frac{1}{3V_0}, \qquad \{\delta K_a^i(x), \delta E_j^b(y)\} = \delta^3(x,y)\delta_a^b\delta_j^i , \tag{21}$$

where the background variables are defined by

$$\bar{p} = \frac{1}{3V_0}\int E_i^a\delta_a^i d^3x, \quad \bar{k} = \frac{1}{3V_0}\int K_a^i\delta_i^a d^3x . \tag{22}$$

Here V_0 is some artificial finite volume.

In this paper, the matter part is represented by a scalar field φ. Similarly, we split the field φ and its conjugate momentum π into homogeneous part and inhomogeneous one as well

$$\varphi = \bar{\varphi} + \delta\varphi, \quad \pi = \bar{\pi} + \delta\pi \; . \tag{23}$$

Hence, the basic Poisson brackets are reduced into

$$\{\bar{\varphi}, \bar{\pi}\} = \frac{1}{V_0}, \quad \{\delta\varphi(x), \delta\pi(y)\} = \delta^3(x-y). \tag{24}$$

For simplicity, we introduce the LQC formalism with finite cell V_0 rather than the whole R^3 region in above description. But the unphysical feature of V_0 can be remedied by lattice refinement model [20]. Since our following calculation only involves $\delta_{\rm inv}$ we adopt the lattice refinement parametrization procedure in [19] to eliminate the effect of artificial volume V_0.

B. The second order Hamiltonian

According to [14], the quantum corrected second order Hamiltonian constraint can conveniently be written as

$$\begin{aligned}
\mathscr{H}^{(2)} &= \mathscr{H}^{(2)}_{\rm grav}[\bar{N}] + \mathscr{H}^{(2)}_{\rm grav}[\delta N] + \mathscr{H}^{(2)}_{\rm matter}[\bar{N}] + \mathscr{H}^{(2)}_{\rm matter}[\delta N] \\
&= \frac{1}{2}\int_\Sigma d^3x \bar{N}\bar{\alpha}\mathfrak{H}^{(2)}_{\rm grav} + \frac{1}{2}\int_\Sigma d^3x \delta N \bar{\alpha}\mathfrak{H}^{(1)}_{\rm grav} + \int_\Sigma d^3x \bar{N}\left[\bar{\nu}\mathfrak{H}^{(2)}_\pi + \bar{\theta}\mathfrak{H}^{(2)}_\nabla + \mathfrak{H}^{(2)}_\varphi\right] \\
&+ \int_\Sigma d^3x \delta N [\bar{\nu}\mathfrak{H}^{(1)}_\pi + \mathfrak{H}^{(1)}_\varphi] \; ,
\end{aligned} \tag{25}$$

where

$$\mathfrak{H}^{(1)}_{\rm grav} = -4(1+f)\bar{k}\sqrt{\bar{p}}\delta^c_j\delta K^j_c - (1+g)\frac{\bar{k}^2}{\sqrt{\bar{p}}}\delta^j_c\delta E^c_j + \frac{2}{\sqrt{\bar{p}}}\partial_c\partial^j\delta E^c_j \ , \tag{26}$$

$$\begin{aligned}\mathfrak{H}^{(2)}_{\rm grav} &= \sqrt{\bar{p}}\delta K^j_c\delta K^k_d\delta^c_k\delta^d_j - \sqrt{\bar{p}}(\delta K^j_c\delta^c_j)^2 - \frac{2\bar{k}}{\sqrt{\bar{p}}}\delta E^c_j\delta K^j_c \\ &\quad -\frac{\bar{k}^2}{2\bar{p}^{3/2}}\delta E^c_j\delta E^d_k\delta^k_c\delta^j_d + \frac{\bar{k}^2}{4\bar{p}^{3/2}}(\delta E^c_j\delta^j_c)^2 - (1+h)\frac{\delta^{jk}}{2\bar{p}^{3/2}}(\partial_c\delta E^c_j)(\partial_d\delta E^d_k) \ ,\end{aligned} \tag{27}$$

$$\mathfrak{H}^{(1)}_{\pi} = (1+f_1)\frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} - (1+f_2)\frac{\bar{\pi}^2}{2\bar{p}^{3/2}}\frac{\delta^j_c\delta E^c_j}{2\bar{p}} \ , \tag{28}$$

$$\mathfrak{H}^{(1)}_{\nabla} = 0 \ , \tag{29}$$

$$\mathfrak{H}^{(1)}_{\varphi} = \bar{p}^{3/2}\left((1+f_3)V_{,\varphi}(\bar{\varphi})\delta\varphi + V(\bar{\varphi})\frac{\delta^j_c\delta E^c_j}{2\bar{p}}\right) \ , \tag{30}$$

$$\mathfrak{H}^{(2)}_{\pi} = (1+g_1)\frac{\delta\pi^2}{2\bar{p}^{3/2}} - (1+g_2)\frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}}\frac{\delta^j_c\delta E^c_j}{2\bar{p}} + \frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^{3/2}}\left((1+g_3)\frac{(\delta^j_c\delta E^c_j)^2}{8\bar{p}^2} + \frac{\delta^k_c\delta^j_d\delta E^c_j\delta E^d_k}{4\bar{p}^2}\right) \ , \tag{31}$$

$$\mathfrak{H}^{(2)}_{\nabla} = \frac{1}{2}(1+g_5)\sqrt{\bar{p}}\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi \ , \tag{32}$$

$$\mathfrak{H}^{(2)}_{\varphi} = \bar{p}^{3/2}\left[(1+g_6)\frac{1}{2}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2 + V_{,\varphi}(\bar{\varphi})\delta\varphi\frac{\delta^j_c\delta E^c_j}{2\bar{p}} + V(\bar{\varphi})\left(\frac{(\delta^j_c\delta E^c_j)^2}{8\bar{p}^2} - \frac{\delta^k_c\delta^j_d\delta E^c_j\delta E^d_k}{4\bar{p}^2}\right)\right] \ , \tag{33}$$

where the definitions of the counterterms can be found in [15] or in the Appendix B of [14]. And the perturbed second order diffeomorphism constraint can be expressed as

$$\begin{aligned}\mathscr{D}^{(2)}[\delta N^a] &= \mathscr{D}^{(2)}_{\rm grav}[\delta N^a] + \mathscr{D}^{(2)}_{\rm matter}[\delta N^a] \\ &= \int_\Sigma d^3x\delta N^a\Big[\bar{p}\partial_a(\delta^d_k\delta K^k_d) - \bar{p}(\partial_k\delta K^k_a) - \bar{k}\delta^k_a(\partial_d\delta E^d_k) + (\bar{\pi}\partial_a\delta\varphi)\Big] \ .\end{aligned} \tag{34}$$

Based on this corrected Hamiltonian, we can get the homogeneous and inhomogeneous part of matter field as

$$\bar{\pi} = \bar{\varphi}'\bar{p}/\bar{\nu}, \delta\pi = \bar{p}\left\{\Big[\delta\varphi' - \bar{\varphi}'(1+f_1)\phi\Big](1-g_1) + \bar{\varphi}'\frac{\delta E^a_i\delta^i_a}{2\bar{p}}\right\}/\bar{\nu}, \tag{35}$$

where f_1 and g_1 are the counterterms.

C. Power spectrum

Calculations can be simplified greatly in the spatially flat gauge ($\psi = 0$, $E = 0$), because the perturbed triad vanishes ($\delta E^a_i = 0$) in this gauge. Here we fix the gauge after having

put quantum corrections in Hamiltonian and having checked consistency. In contrast, in references [39], the authors fixed the gauge beforehand. We believe our treatment is more consistent. By solving the constraint equations, the perturbed lapse function and shift vector read

$$\phi = \frac{1}{2}\frac{\bar{\alpha}}{\bar{\nu}}\frac{\bar{\varphi}'}{\mathcal{H}}\frac{1}{1+f}\delta\varphi, \tag{36}$$

$$\Delta B = -\frac{1}{2}\frac{\bar{\alpha}}{\bar{\nu}}\frac{1}{\mathcal{H}}\frac{1+f_3}{1+f}\{\bar{\varphi}'\delta\varphi' - \bar{\varphi}'^2(1+f_1)\phi + \bar{\nu}\bar{p}V_{,\varphi}(\bar{\varphi})\delta\varphi\} - 3\mathcal{H}(1+f)\phi\,. \tag{37}$$

And the extrinsic curvature is

$$\bar{\alpha}\delta K^i_a = -\delta^i_a\mathcal{H}(1+f)\phi - \partial_a\partial^i B, \tag{38}$$

where $\bar{\alpha}\bar{k} = \mathcal{H}$.

In the spatially flat gauge, the total second order Hamiltonian becomes

$$\begin{aligned}\mathscr{H}^{(2)} &= \int_\Sigma d^3x\Big\{\Big[\frac{3\bar{p}\bar{\alpha}}{2\bar{\nu}^2} - \frac{\bar{\alpha}^2\bar{p}}{4\bar{\nu}^3}\frac{\bar{\varphi}'^4}{\mathcal{H}^2}(1-g_1+2f_1-2f) \\ &+ \frac{(1+g_6)}{2\bar{p}^2}V_{,\varphi\varphi}(\bar{\varphi}) + \frac{\bar{\alpha}\bar{p}^2}{\bar{\nu}}\frac{\bar{\varphi}'}{\mathcal{H}}V_{,\varphi}(\bar{\varphi})(1+f_3-f)\Big]\delta\varphi^2 \\ &+ \frac{\bar{p}}{2\bar{\nu}}(1-g_1)\delta\varphi'^2 + \frac{\bar{p}\bar{\theta}}{2}(1+g_5)\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi\Big\}\,,\end{aligned} \tag{39}$$

where $\bar{\theta}$ is also a correction function for the inverse volume and $\bar{\alpha}^2 = \bar{\nu}\bar{\theta}$. In this gauge, the dynamical inflaton perturbation $\delta\varphi$ coincides with the Sasaki-Mukhanov variable $u = z\zeta$, with

$$z = \frac{\varphi'}{\mathcal{H}}\left[1 + \left(\frac{\alpha_0}{2} - \nu_0\right)\delta_{\rm inv}\right]\,. \tag{40}$$

Then, one can derive the Mukhanov equation [19]

$$u'' - (c_s^2\Delta + \frac{z''}{z})u = 0\,, \tag{41}$$

where c_s is the propagation speed of the perturbation. The solution of the above equation is [19]

$$\begin{aligned}u(k,\tau) &= \frac{H}{\sqrt{2k^3}}e^{-ik\tau}\Big[1 + ik\tau - \frac{\chi}{2(\sigma+1)}(1+ik\tau)\delta_{\rm inv}\Big]\,, \\ &= \frac{H}{\sqrt{2k^3}}e^{-ik\tau}\left[F\left(\frac{k_0}{k}\right) + ikF\left(\frac{k_0}{k}\right)\tau + \mathcal{O}(k^2\tau^2)\right]\,,\end{aligned} \tag{42}$$

where

$$F\left(\frac{k_0}{k}\right) = \left[1 - \frac{\chi}{2(\sigma+1)}\delta_{\rm inv}\right] = \left[1 + C\left(\frac{k_0}{k}\right)^{\sigma}\right], \tag{43}$$

where $\chi = \sigma\nu_0(1+\sigma/6)/3 + \alpha_0(5-\sigma/3)/2$, $C = -\frac{\delta_0\chi}{2(\sigma+1)}$ and $\delta_0 = \delta(k_0)/\alpha_0$. The latter variable $\delta(k_0)$ is constrained by the cosmic observational data [21], such as Cosmic Microwave Background and Large Scale Structures. For the specific inflationary models with a quadratic potential and $\sigma = 2$, $\delta(k_0) \sim \mathcal{O}(10^{-5})$. In this paper we take $\alpha_0 \sim \mathcal{O}(10^{-2})$, i.e., the variable δ_0 is of the order $\mathcal{O}(10^{-3})$. Moreover, Eq. (42) tells us that, on the one hand the inverse volume corrections become important for long wave modes with $k \ll k_0$; on the other hand long wave modes cross horizon earlier than the short ones. It means that the inverse volume correction will leave more hints on large scales than small ones. These features are much different from those of the inflationary models with higher derivative terms such as K-inflation in Einstein gravity.

Using the canonical quantization, we have

$$\zeta(\vec{k},\tau) = \zeta^+ + \zeta^- = \zeta(\vec{k},\tau)a_{\vec{k}} + \zeta^*(\vec{k},\tau)a^\dagger_{-\vec{k}}, \tag{44}$$

where $\zeta(\vec{k},\tau) = u(\vec{k},\tau)/z(\vec{k})$. Then the two-point correlation functions of curvature perturbations can be calculated straight forwardly as

$$\langle\zeta_{\vec{k}}\zeta_{\vec{k}'}\rangle = \frac{|u|^2}{z^2} \approx (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{H^2}{2\epsilon_0 k^3}\{1+\gamma_s\delta_{\rm inv}\}, \tag{45}$$

with

$$\gamma_s = \nu_0(1+\sigma/6) + \sigma\alpha_0/2\epsilon - \chi/(\sigma+1) \tag{46}$$

where the first and second terms in (46) comes from the $z(\vec{k})$ factor in the gauge transformations, and the third term attributes to the modifications of vacuum state (42).

Finally, we can get the primordial power spectrum of curvature perturbations as

$$P_\zeta(k) \equiv \frac{k^3}{2\pi^2}\langle\zeta^2\rangle \approx \frac{H^2}{4\pi^2\epsilon_0}(1+C\frac{k_0}{k}). \tag{47}$$

This result is in agreement with that in [21]. When all the corrections vanish, this result is back to the case of the single field inflationary model in Einstein gravity [30–34]. The primordial power spectrum of curvature perturbations and angular power spectrum are plotted in Fig. 1, where the dotted (purple), dashed (deep blue) and the solid (light blue) curves correspond to different values of the parameter C ($C = 0, 4\times10^{-4}, 3\times10^{-3}$) which was defined after Eq.(43).

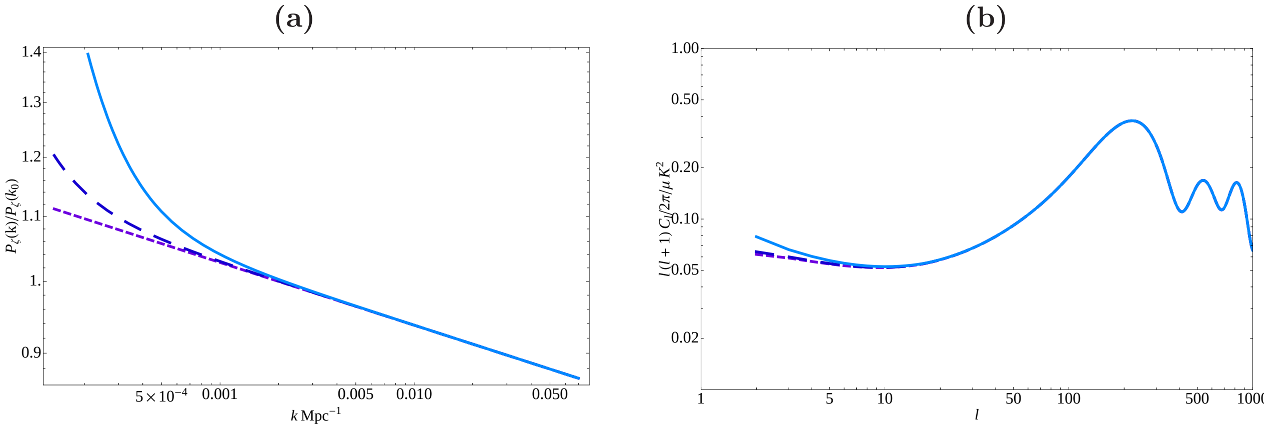

FIG. 1: Primordial $P_\zeta(k)$ (a) and angular C_l (b) power spectra. Here, we set the slow-roll parameters as the typical values $\epsilon_0 = \eta_0 = 0.01$ and $\sigma = 2$. In our calculations, the pivot wavenumber k_0 equals to 0.002Mpc^{-1}.

IV. BISPECTRUM

In this section, we firstly derive the third order Hamiltonian in the spatially flat gauge; then calculate the three-point functions of the primordial curvature perturbations; and finally figure out the sizes and shapes of the bispectrum.

A. The third order Hamiltonian and In-In formalism

We can get the corrected third order Hamiltonian

$$\begin{aligned} \mathscr{H}^{(3)} &= \mathscr{H}^{(3)}_{\text{grav}}[\bar{N}] + \mathscr{H}^{(3)}_{\text{grav}}[\delta N] + \mathscr{H}^{(3)}_{\text{matter}}[\bar{N}] + \mathscr{H}^{(3)}_{\text{matter}}[\delta N] \\ &= \frac{1}{2}\int_\Sigma d^3x \bar{N}\bar{\alpha}\mathfrak{H}^{(3)}_{\text{grav}} + \frac{1}{2}\int_\Sigma d^3x \delta N \bar{\alpha}\mathfrak{H}^{(2)}_{\text{grav}} + \int_\Sigma d^3x \left[\bar{\nu}\mathfrak{H}^{(3)}_\pi + \bar{\theta}\mathfrak{H}^{(3)}_\nabla + \mathfrak{H}^{(3)}_\varphi\right] \\ &+ \int_\Sigma d^3x \delta N[\bar{\nu}\mathfrak{H}^{(2)}_\pi + \mathfrak{H}^{(2)}_\varphi + \bar{\theta}\mathfrak{H}^{(2)}_\nabla] , \end{aligned} \tag{48}$$

where the expressions for $\mathfrak{H}^{(3)}_{grav}$, $\mathfrak{H}^{(3)}_\pi$, $\mathfrak{H}^{(3)}_\nabla$, $\mathfrak{H}^{(3)}_\varphi$ are complicated, and we list them in Appendix A.

The perturbed third order diffeomorphism constraint is

$$\mathscr{D}^{(3)}[\delta N^a] := \frac{1}{\gamma}\int_\Sigma d^3x\delta N^a\Big[\frac{1}{2\bar{p}}\epsilon_b^{ij}(\partial_a\partial_c\delta E_i^c)\delta E_j^b + \gamma\partial_a\delta K_b^j\delta E_j^b - \frac{1}{2\bar{p}}\epsilon_a^{jk}(\partial_b\partial_c\delta E_k^c)\delta E_j^b - \gamma\partial_b\delta K_a^j\delta E_j^b - \frac{1}{2\bar{p}}\epsilon_a^{jk}\partial_c\delta E_k^c\partial_b\delta E_j^b - \gamma\delta K_a^j\partial_b\delta E_j^b + \delta\pi\partial_a\delta\varphi\Big]. \tag{49}$$

We calculate the non-Gaussianity in the interaction picture [35]

$$\langle\zeta^3(\tau)\rangle = \langle U_{\rm int}^{-1}\zeta^3(\tau)U_{\rm int}(\tau,\tau_0)\rangle, \quad U_{\rm int} = e^{-i\int_{\tau_0}^{\tau}\mathscr{H}_{\rm int}(\tau')d\tau'}. \tag{50}$$

Up to the first order, we have

$$\langle\zeta^3(\tau)\rangle = -i\int_{\tau_0}^{\tau} d\tau'\langle[\zeta^3(\tau),\mathscr{H}_{\rm int}(\tau')]\rangle. \tag{51}$$

B. Sizes and shapes

Based on the second order anomaly free perturbative LQC theory, we combine Eq.(48) with Eq.(49) and arrive at the third order interaction Hamiltonian with some counterterms

$$\begin{aligned}\mathscr{H}_{\rm int}^{(3)} = \mathscr{H}^{(3)} + \mathscr{D}^{(3)} = \int d^3x\Big[&\frac{\bar{p}}{2\bar{\nu}}(1-g_1)\delta\varphi'^2\phi + \frac{\bar{p}\bar{\theta}}{2}(1+g_5)\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi\phi \\ &+\frac{\bar{p}}{\bar{\nu}}(1-g_1)(\partial^a B)\delta\varphi'\partial_a\delta\varphi - \frac{3\bar{p}}{\bar{\alpha}}(1+2f)\mathcal{H}^2\phi^3 \\ &+\frac{\bar{p}}{2\bar{\nu}}(1+2f_1-g_1)\bar{\varphi}'^2\phi^3 + \frac{(1+g_6)\bar{p}^2}{2}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2\phi \\ &+\frac{\bar{p}^2}{6}V_{,\varphi\varphi\varphi}(\bar{\varphi})\delta\varphi^3 - \frac{(1+f_1-g_1)\bar{p}}{\bar{\nu}}\bar{\varphi}'\delta\varphi'\phi^2 + ...\Big],\end{aligned} \tag{52}$$

where the first three terms are in the leading order under the slow-roll approximation. In the following calculations, we only consider these terms. Comparing Eq. (52) with those in [30–34], we could attribute two kinds of modifications in the interaction Hamiltonian to the inverse volume corrections. One comes from the modification of vacuum state (42), the other from the background dependent coefficients, such as $(\bar{\nu},\bar{\theta},\bar{\alpha},g_1,\cdots)$. However, the non-Gaussian signatures from the latter are contaminated greatly by the cosmic variance. Hence, we ignore the modifications in $\bar{\nu}$ *etc.*, when we calculate the parts of three-point functions directly from the In-In formalism. In another words, we ignore the quantum anomaly behavior for the Hamiltonian in this work. In order to demonstrate the reasons

more manifestly, we take the first term in (52) as an example.

$$
\begin{aligned}
&\int d^3x \frac{\bar{p}}{2\bar{\nu}}(1-g_1)\delta\varphi'^2\phi \\
=& \int d^3x \frac{\bar{\varphi}'}{\mathcal{H}}\frac{\bar{p}}{4}\delta\varphi'^2\delta\varphi + \int d^3x \frac{\bar{\varphi}'}{\mathcal{H}}\frac{\bar{p}}{4}\Big[\frac{5\alpha}{2} - 2\nu - \nu\left(\frac{\sigma}{3}+1\right)\Big]\delta_{\rm inv}\delta\varphi'^2\delta\varphi \ , \qquad (53)
\end{aligned}
$$

where the first term in the second line appears in the usual form, while the second one contains $\delta_{\rm inv} = \delta_{max}\tau^{\sigma}$. Note $\delta_{\rm inv}$ depends on k, there is a corresponding $\bar{k}$ related to δ_{max}. When we put these terms into the formalism (51) and perform the time integral, we will obtain such corrections

$$
\frac{\bar{k}^2}{K^3}\mathcal{G}(k_1,k_2,k_3) + \cdots \ , \qquad K \equiv k_1 + k_2 + k_3 \ , \qquad (54)
$$

where the dots denotes higher order corrections. From the above expression, we can see that such term peaks at $\bar{k} \gg K$. Since larger k gives smaller $\delta_{\rm Pl}$, and quadruple is the lowest detectable mode in CMB, $\bar{k}$ corresponds to $\bar{l} = 2$. So Eq.(54) peaks on the very low $l \ll 2$ region where the cosmic variance dominates over the signals (See Figure 1 and Figure 2 in [21]). Although the non-Gaussian features are presented on both large scale and small scale, above analysis implies that we can ignore the effects on small scales. Hence, we can ignore such terms in our following calculations. Once we ignore the corrections in the background dependent coefficients in Eq.(52), the form of the third order interaction Hamiltonian reduces to the usual one [30–34].

In order to eliminate the terms proportional to the linear equations, we need to do the field redefinition

$$
\begin{aligned}
\zeta &= \zeta_c - \frac{1}{2}\left(1 - \frac{\bar{\varphi}''}{\bar{\varphi}'\mathcal{H}}\right)\zeta_c^2 + \frac{1}{8}\frac{\bar{\varphi}'^2}{\mathcal{H}^2}\zeta_c^2 + \frac{1}{4}\frac{\bar{\varphi}'^2}{\mathcal{H}^2}\partial^{-2}(\zeta_c\partial^2\zeta_c) \ , \\
&= \zeta_c + C_1\zeta_c^2 + C_2\partial^{-2}(\zeta_c\partial^2\zeta_c) \ . \qquad (55)
\end{aligned}
$$

Then, the interaction Hamiltonian can be reduced into the simple form

$$
\mathscr{H}^{(3)}_{\rm int} \simeq -\int d^3x \frac{\bar{\varphi}'^4}{\mathcal{H}^3}\bar{p}\zeta_c'^2\partial^{-2}\zeta_c' + \cdots \ . \qquad (56)
$$

According to [30–34], after a field redefinition of the schematic form $\zeta = \zeta_c + \lambda\zeta_c^2$ then the correlation function will contain two terms

$$
\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle = \langle\zeta_c(x_1)\zeta_c(x_2)\zeta_c(x_3)\rangle + 2\lambda\Big[\langle\zeta(x_1)\zeta(x_2)\rangle\langle\zeta(x_1)\zeta(x_3)\rangle + \text{cyclic}\Big] \ , \quad (57)
$$

where the first term can be computed by the In-In formalism and the second term comes from the field redefinition $\zeta = \zeta_c + \lambda\zeta_c^2$.

Let us firstly calculate the first term

$$
\begin{aligned}
\langle\zeta_c(\tau,k_1)\zeta_c(\tau,k_2)\zeta_c(\tau,k_3)\rangle &= -i\int_{-\infty}^{0} d\tau\langle 0|\left[\zeta_c(\tau,k_1)\zeta_c(\tau,k_2)\zeta_c(\tau,k_3),\mathcal{H}^{(3)}_{\rm int}(\tau)\right]|0\rangle \\
&= \frac{(2\pi)^3\delta(\sum\vec{k}_i)H^6}{\left[\prod_{i=1}^{3}z^2(k_i)2k_i^3\right]}\frac{\dot{\bar{\varphi}}^4}{H^4}\frac{4}{H^2}\frac{\sum_{i>j}k_i^2k_j^2}{K}\left\{\prod_{j=1}^{3}\left[1+C\left(\frac{k_0}{k_j}\right)^{\sigma}\right]^2\right\}\,,
\end{aligned}
\tag{58}
$$

where $z(k)$ and C are defined in Eqs. (40) and (43), respectively. Here, we emphasize again that corrections from the background dependent coefficients are neglected in the above expressions. However, in the next calculations of the parts from field redefinition, such corrections should be taken into account for consistence. Because the background dependent coefficients $(\bar{\varphi}'/\mathcal{H}|_*\,,\cdots)$ take the value at the moment when the corresponding mode crosses horizon $(\tau_*\sim k^{-1})$, they contain corrections such as $(k_0/k_i)^{\sigma}$. Of course, these terms also peak at the points where $k_0 \gg k_i$, they become important, particularly, in the squeezed triangle limit $(k_1 \ll k_2, k_3)$.

The contributions from field redefinition can be decomposed into two parts, one is

$$
(2\pi)^3\delta(\sum\vec{k}_i)\frac{4H^4}{\left[\prod_{i=1}^{3}z^2(k_i)2k_i^3\right]}\left[\sum_{i=1}^{3}C_1(k_i)k_i^3z^2(k_i)\right]\,,
\tag{59}
$$

and the other is

$$
(2\pi)^3\delta(\sum\vec{k}_i)\frac{2H^4}{\left[\prod_{i=1}^{3}z^2(k_i)2k_i^3\right]}\left[\sum_{i\neq j}C_2(k_i)z^2(k_i)k_ik_j^2\right]\,,
\tag{60}
$$

where $H = \frac{\dot{\bar{p}}}{2\bar{p}}$ and the overdot stands for derivative with respect to the cosmic time. The $C_1(k_i), C_2(k_j)$ terms can be read from Eq.(55) by substituting τ with k_i^{-1}.

In summary, we conclude that the forms of interaction Hamiltonian are exactly the same as the usual one [30–34], however, the inverse volume corrections $\delta_{\rm inv}$ will make some contribution to the bispectrum. There are mainly two sources, one is the modifications to the standard Bunch-Davies vacuum, the other comes from the $z(k)$ factor in the gauge transformation $\zeta(k) = u(k)/z(k)$. Furthermore, we can expand the above results in terms of $\delta_{\rm inv} = \delta_0(k_0/k_i)^{\sigma}$ and obtain

$$
(2\pi)^3\delta(\sum\vec{k}_i)\left[\mathcal{F}_{\rm single}(k_1,k_2,k_3)+\mathcal{F}_1(k_1,k_2,k_3)+\mathcal{F}_2(k_1,k_2,k_3)\right]\,,
\tag{61}
$$

where

$$\mathcal{F}_{\rm single} = \frac{(2\pi)^4 P_\zeta^2}{4\Big[\prod_{j=1}^3(2k_j^3)\Big]}\left\{(3\epsilon_0 - 2\eta_0)\sum_i k_i^3 + \epsilon_0\sum_{i\neq j} k_i k_j^2 + 8\epsilon_0\frac{\sum_{i>j}k_i^2k_j^2}{K}\right\} , \tag{62}$$

is the usual leading term and

$$\mathcal{F}_1 = \left[\omega_1\sum_{l=1}^3\left(\frac{k_0}{k_l}\right)^\sigma\right]\mathcal{F}_{\rm single} , \tag{63}$$

$$\mathcal{F}_2 = \frac{(2\pi)^4 P_\zeta^2}{4\Big[\prod_{j=1}^3(2k_j^3)\Big]}\left\{\Big[2\omega_3'(\epsilon_0-\eta_0)+\omega_2'\epsilon_0\Big]\sum_i k_i^3\left(\frac{k_0}{k_i}\right)^\sigma + \omega_2'\epsilon_0\sum_{i\neq j}k_j^2k_i\left(\frac{k_0}{k_i}\right)^\sigma\right\} , \tag{64}$$

are the inverse volume correction terms, and here the relevant coefficients are

$$\omega_1 = 2C - C_z , \tag{65}$$

$$\omega_2' = \gamma_\epsilon + C_z - 2C , \tag{66}$$

$$\omega_3' = \gamma_\eta + C_z - 2C , \tag{67}$$

$$C_z = -\delta_0\left[\nu_0\left(\frac{\sigma}{6}+1\right)+\frac{\sigma\alpha_0}{2\epsilon_0}\right] , \tag{68}$$

$$C = -\frac{\delta_0\chi}{2(\sigma+1)} . \tag{69}$$

Particularly, we figure out that the corrections from $\Big[\prod_{i=1}^3 z^2(k_i)\Big]$ terms in the denominators in Eqs.(58), (59) and (60) are absorbed into $\mathcal{F}_1$ shape, while all other corrections are collected in $\mathcal{F}_2$. In the next section, we will find $\mathcal{F}_1$ shape provides an unique signal from LQC mechanism, and more importantly, this signal is independent of the inflationary models, because $\Big[\prod_{i=1}^3 z^2(k_i)\Big]$ terms always appear in the gauge transformations (40). Namely it is an universal signal in the LQC scenario.

The single shape is the usual one, while $\mathcal{F}_1$ and $\mathcal{F}_2$ arise only in LQC scenarios. Sizes of the two new parts are proportional to parameters (ω_1 , $\cdots$), which are in the order of $\mathcal{O}(10^{-3},10^{-4})$. That is to say that, these new non-Gaussian features from LQC are smaller than those of usual inflationary models in Einstein gravity by a factor at least 10^{-3}. Although this does be a tiny number, considering these features sourced by quantum effect, this factor is not small as initially expected. Furthermore, as stated above, because the inverse volume

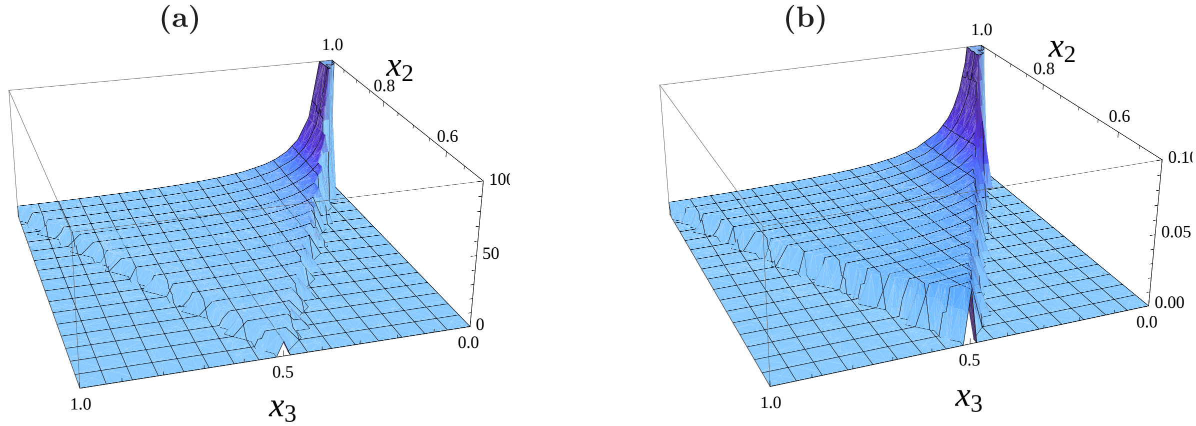

FIG. 2: Single (a) and $\mathcal{F}_1$ (b) shapes. The z-axis is $x_2^2 x_3^2 \mathcal{F}(x_2, x_3)/P_\zeta^2$, here we set $x_1 = 1$. The slow-roll parameters take the typical values $\epsilon_0 = \eta_0 = 0.01$. Here and hereafter we set $\sigma = 2$.

corrections in the interaction Hamiltonian can be neglected, i.e. we could use the usual Hamiltonian directly in the In-In formalism by substituting the Bunch-Davies vacuum state with the one given in Eq.(42), we argue that the expectations on the sizes of bispectrum should hold for any inflationary models in LQC scenario.

In Fig. 2 (a) (b) and Fig. 3 (c), we plot $x_1^2 x_2^2 x_3^2 \mathcal{F}(x_1, x_2, x_3)/P_\zeta^2$, with $x_i \equiv k_i/k_1$. The difference between shapes $\mathcal{F}_1$ and $\mathcal{F}_2$ is plotted in Fig. 3 (d). We can see that all the three shapes peak at the squeezed limit ($x_2 = 1, x_3 = 0$), however, the substructures are different among them. Compared to the single shape, $\mathcal{F}_1$ shape upraises at the corner ($x_2 = 0, x_3 = 0.5$), while $\mathcal{F}_2$ flattens at the same point. From Fig. 3 (d), we can also see that $\mathcal{F}_2$ peaks more dramatically in the squeezed corner than $\mathcal{F}_1$.

V. SHAPE CORRELATIONS

In the Figures 2 and 3, all the three shapes looks similar. In order to figure out the differences among shapes more quantitatively, we need to calculate the correlations between them.

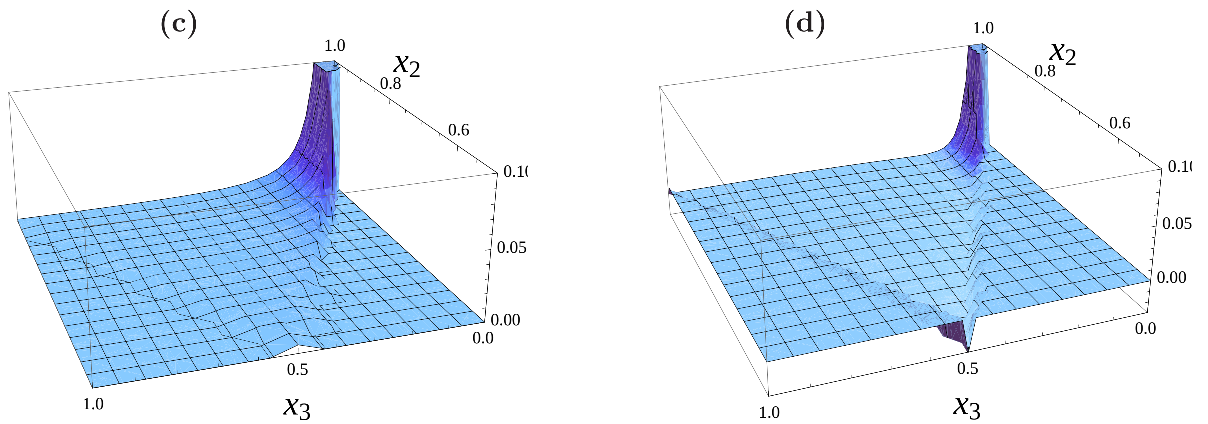

FIG. 3: $\mathcal{F}_2$ (c) shape and $\mathcal{F}_2 - \mathcal{F}_1$ (d). The coefficients $(\omega_1\ , \omega_2'\ , \omega_3')$ in $\mathcal{F}_1, \mathcal{F}_2$ are set unity.

Firstly, we define a 3D shape function [40–42]

$$S(k_1, k_2, k_3) = \frac{1}{N}(k_1 k_2 k_3)^2 \mathcal{F}(k_1, k_2, k_3)\ , \tag{70}$$

where N is a normalization factor which will not affect the following calculations.

Then, we construct the products of two shape functions

$$F(S, S') = \int_{\mathcal{V}_k} S(k_1, k_2, k_3) S'(k_1, k_2, k_3) \omega(k_1, k_2, k_3) d\mathcal{V}_k\ , \tag{71}$$

where $\omega(k_1, k_2, k_3)$ is a weight function and $\mathcal{V}_k$ is the integration domain constrained by the triangle inequality.

Finally, we arrive at the 3D shape function correlator

$$\bar{\mathcal{C}}(S, S') = \frac{F(S, S')}{\sqrt{F(S, S) F(S', S')}}\ . \tag{72}$$

This quantity describe the cross correlations between two different shapes.

Furthermore, for a large series of well-motivated shapes, the above descriptions can be simplified. One can define the distance described by k from the origin of (k_1, k_2, k_3)-momentum space to the particular triangle slice which is perpendicular to $(1, 1, 1)$ direction,

$$k \equiv \frac{1}{2}(k_1 + k_2 + k_3)\ , \tag{73}$$

then, we introduce another two new variables

$$
\begin{aligned}
k_1 &= k(1-\beta) , &(74)\\
k_2 &= \frac{1}{2}k(1+\alpha+\beta) , &(75)\\
k_3 &= \frac{1}{2}k(1-\alpha+\beta) . &(76)
\end{aligned}
$$

In the domain constrained by the triangle inequality, $0 \le k \le \infty$, $0 \le \beta \le 1$ and $-(1-\beta) \le \alpha \le 1-\beta$. For the classes of models with homogeneous shape, which means that the powers of wavenumber in shapes are homogeneous, the k dependence in the 3D shape function can be separated

$$S(k_1, k_2, k_3) = f(k)\mathscr{S}(\alpha, \beta) , \qquad d\mathcal{V}_k = dk_1 dk_2 dk_3 = k^2 dk d\alpha d\beta . \tag{77}$$

In fact, for the models considered in [40–42] and here, $f(k) \propto$ const. Hence, we can focus on the 2D shape function $\mathscr{S}(\alpha, \beta)$ and the integral over k cancels when one calculates the shape function correlators.

Further, one can introduce

$$\alpha = 1 - x , \qquad \beta = xy/3 , \quad (0 \le x, y \le 1) , \tag{78}$$

to square the integration regime. By using these variables, the integral measurement becomes

$$d\alpha d\beta = x dx dy . \tag{79}$$

From the above expression, we can read the weight function $w(x, y) = x$. This choice works well for all the shapes mentioned in [40–42], however, for our new shapes $\mathcal{F}_1$ and $\mathcal{F}_2$ the correlation matrices c_{mn} defined in (84), suffers from divergence. In our calculation we therefore use a new variable $\xi^2 = x$

$$d\alpha d\beta = \xi^3 d\xi dy , \tag{80}$$

to eliminate such divergence. Using variables (ξ, y), we can decompose the shape $\mathscr{S}(\xi, y)$ on any triangle slice with analogous radial polynomials $R_m(\xi)$ and shifted Legendre polynomials $\bar{P}_n(y)$

$$\mathscr{S}(\xi, y) = \sum_{m,n} c_{mn} R_m(\xi) \bar{P}_n(y) , \tag{81}$$

where the first few $R_m(\xi)$ eigenfunctions are

$$R_0 = \sqrt{2}\,, \quad R_1 = \sqrt{4}(-2+3\xi)\,, \quad R_2 = \sqrt{6}(3-12\xi+10\xi^2)\,,$$
$$R_3 = \sqrt{8}(-4+30\xi-60\xi^2+35\xi^3)\,, \cdots\,. \tag{82}$$

And $\bar{P}_n(y)$ eigenfunctions are

$$\bar{P}_0 = 1\,, \quad \bar{P}_1 = \sqrt{3}(-1+2y)\,, \quad \bar{P}_2 = \sqrt{5}(1-6y+6y^2)\,,$$
$$\bar{P}_3 = \sqrt{7}(-1+12y-30y^2+20y^3)\,, \cdots\,. \tag{83}$$

Thus, one can define the correlation matrix c_{mn}

$$c_{mn} = \int_0^1 d\xi \int_0^1 dy \xi^3 \mathscr{S}(\xi,y) R_m(\xi) \bar{P}_n(y)\,. \tag{84}$$

The c_{mn} matrices for local, equilateral, single field model, $\mathcal{F}_1$ and $\mathcal{F}_2$ are listed in (85) and Fig. 4. The explicit definitions of 2D shape function $\mathscr{S}(\xi,y)$ can be found in Appendix B.

$$\begin{pmatrix} 1.00 & -0.14 & 0.03 & 0.00 \\ 0.38 & -0.07 & 0.02 & 0.00 \\ 0.04 & -0.01 & 0.01 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \end{pmatrix}, \quad \begin{pmatrix} 1.00 & 0.40 & -0.12 & 0.01 \\ 0.68 & 0.27 & -0.09 & 0.01 \\ 0.21 & 0.07 & -0.03 & 0.00 \\ 0.01 & 0.00 & -0.01 & 0.00 \end{pmatrix}, \quad \begin{pmatrix} 1.00 & -0.07 & 0.01 & 0.00 \\ 0.43 & -0.02 & 0.01 & 0.00 \\ 0.07 & 0.00 & 0.00 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \end{pmatrix},$$
$$\begin{pmatrix} 1.00 & -0.19 & 0.04 & 0.00 \\ -0.26 & 0.09 & -0.01 & 0.00 \\ 0.44 & -0.10 & 0.02 & 0.00 \\ -0.32 & 0.08 & -0.01 & 0.00 \end{pmatrix}, \quad \begin{pmatrix} 1.00 & -0.12 & 0.24 & 0.00 \\ 0.30 & -0.03 & 0.01 & 0.00 \\ 0.07 & -0.01 & 0.01 & 0.00 \\ 0.02 & 0.00 & 0.00 & 0.00 \end{pmatrix}. \tag{85}$$

From Fig. 4, we can see that $\mathcal{F}_1$ term differs from others explicitly, while $\mathcal{F}_2$ is almost indistinguishable with the local form visually.

Armed with the above results, one can calculate 2D shape correlator

$$\mathscr{C}(\mathscr{S},\mathscr{S}') \equiv \frac{\mathscr{F}(\mathscr{S},\mathscr{S}')}{\sqrt{\mathscr{F}(\mathscr{S},\mathscr{S})\mathscr{F}(\mathscr{S}',\mathscr{S}')}}\,, \tag{86}$$

where the product is defined through

$$\mathscr{F}(\mathscr{S},\mathscr{S}') \equiv \int_{\mathcal{S}_k} \mathscr{S}(\xi,y)\mathscr{S}'(\xi,y)\xi^3 d\xi dy = \sum_{m,n} c_{mn}c'_{mn}\,. \tag{87}$$

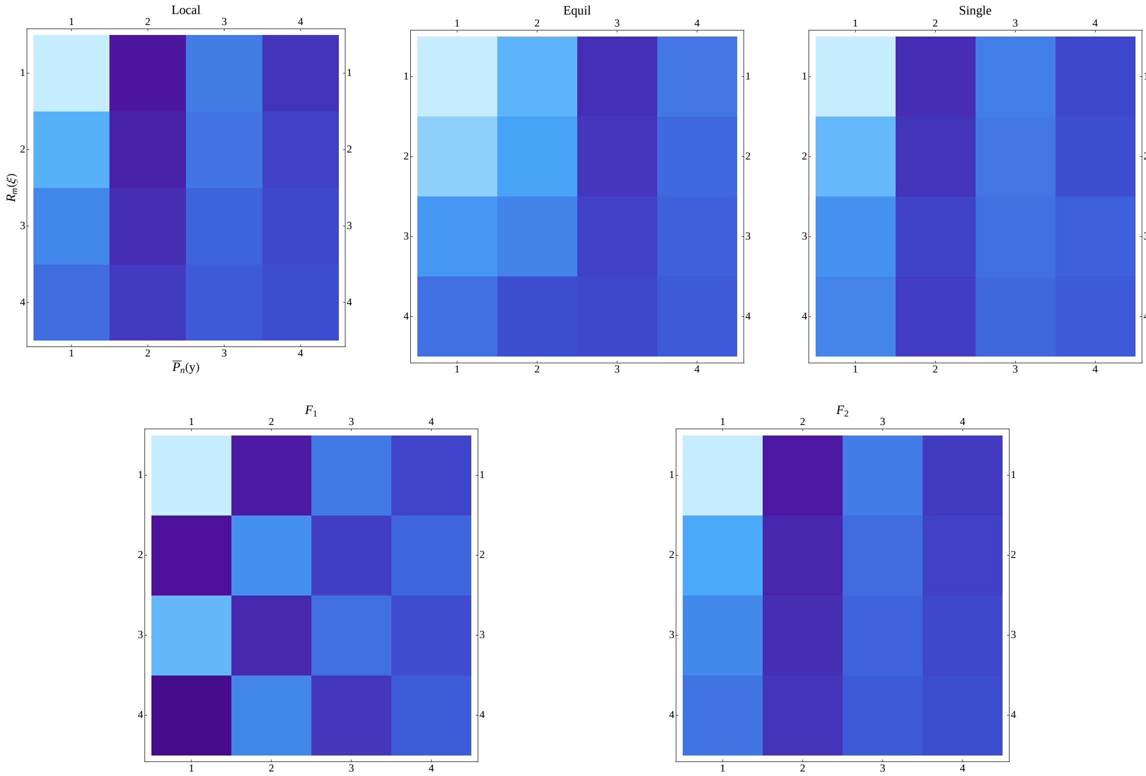

FIG. 4: Correlation matrices in (85). Light blue denotes for +1, and the deep for -1.

	Local	Equi	Single	$\mathcal{F}_1$	$\mathcal{F}_2$
Local	1	0.826	0.996	0.722	0.996
Equil		1	0.874	0.535	0.826
Single			1	0.707	0.992
$\mathcal{F}_1$				1	0.769
$\mathcal{F}_2$					1

TABLE I: 2D shape correlators.

The numerical results of the 2D correlators are listed in Tablet I, from which we can find that the correlations of shape $\mathcal{F}_1$ are low with all other four shapes, while shape $\mathcal{F}_2$ possesses high correlations with local (particularly), equilateral as well as single form. That is to say that $\mathcal{F}_2$ is almost indistinguishable with local form, while $\mathcal{F}_1$ does be the unique signal of LQC. As already argued in the previous section, $\mathcal{F}_1$ is an universal shape in LQC scenario, so we can identify this shape as a new window for LQC scenario.

Finally, let us estimate the parameter $f_{\rm NL}$. Here we focus on the two new shapes $\mathcal{F}_{1,2}$, from

Tab. I we can see that $\mathcal{F}_2$ is highly correlated with the local form, while $\mathcal{F}_1$ is less correlated with them. The contributions to $f_{\rm NL}^{\rm local}$ from these two shapes can be easily estimated as

$$\Delta f_{\rm NL}^{\rm local} \sim \omega \times \mathscr{C}(\mathscr{S}_{\rm local}, \mathscr{S}_{\mathcal{F}_{1,2}}) \times f_{\rm NL}^{\rm local}(\text{single}) \sim \mathcal{O}(10^{-3}) f_{\rm NL}^{\rm local}(\text{single}) \;, \tag{88}$$

where ω represents in short parameters $(\omega_1, \omega_2', \omega_3')$ in (63) and (64), whose typical values are of order $\mathcal{O}(10^{-3})$. Because $f_{\rm NL}^{\rm local}(\text{single})$ in the usual case is of order $\sim \epsilon_0$ [43], the contributions from inverse volume corrections is completely negligible. However, as argued before, our results should be robust for other inflationary models in LQC scenarios, especially those with large non-Gaussianities, such as K-inflation, DBI inflation *etc.* So, one can anticipate that in those models with large non-Gaussianities the features from inverse volume corrections in LQC scenarios might be observed.

VI. CONCLUSIONS

In this paper we investigated the contributions to the cosmic primordial scalar bispectrum from the inverse volume corrections in LQC scenarios. We derived the interaction Hamiltonian, however, we found that the new interactions contribute greatly to the modes with $k_1 + k_2 + k_3 \ll \bar{k}$. Because the scales corresponding to $\bar{k}$ is very large, here we take $\bar{k} \approx 0.00014\text{Mpc}^{-1}$ corresponding to quadruple mode $\bar{l} = 2$, it means that the three wave lengthes in the bispectrum are all on the super-horizon scales. On so large scales the cosmic variance usually dominates over the signals, we hence neglected these new interactions in our calculations. That is to say, the interaction Hamiltonian we used is the same form as the usual one for the single field inflation models in Einstein gravity. This greatly simplifies our calculations, and more importantly, makes our results robust, i.e., our results should hold for other inflationary models in LQC scenarios.

Although the Hamiltonian shares the same forms as the one in Einstein gravity, the inverse volume corrections $\delta_{\rm inv} \propto (k_0/k)^\sigma$ still contribute to the bispectrum. Roughly speaking, there are two aspects, one comes from the deviations from the standard Bunch-Davies vacuum; the other attributes to the non-trivial gauge transformations $\zeta(k) = u(k)/z(k)$ from the spatially flat gauge to the observable curvature perturbations. Consequently, we obtained the three-point functions of the gauge invariant curvature perturbations. We found that, except for the usual single component in slow-roll inflation models, two new shapes arise due

to the corrections, namely $\mathcal{F}_{1,2}$. Furthermore, we performed a careful analysis on the new shapes. We found that, the whole profiles for all the three shapes (single, $\mathcal{F}_{1,2}$) are visually similar, i.e. they peak at the squeezed limit. However, the substructures among them are different. Compared to the single shape, $\mathcal{F}_1$ shape upraises at another corner (See. Fig. 2 and 3), while $\mathcal{F}_2$ flattens at the same point, and $\mathcal{F}_2$ peaks more dramatically in the squeezed corner than $\mathcal{F}_1$. In addition, we investigated the correlations among five shapes, including local, equilateral, single and $\mathcal{F}_{1,2}$. The results show that $\mathcal{F}_2$ is highly correlated with the local type, while $\mathcal{F}_1$ is less. It means that the latter can provide a new window for probing the loop quantum mechanisms using cosmic primordial bispectrum information. Finally, we estimated the order of observable parameter $\Delta f_{\rm NL}^{\rm local} \sim \mathcal{O}(10^{-3}) \times f_{\rm NL}^{\rm local}(\text{inflation})$ from the inverse volume corrections.

The non-Gaussianity from the inverse volume corrections in LQC scenarios is tiny and still undetectable currently, however, considering they are generated by the quantum effect, which is naively expected of the order $\mathcal{O}(\text{GUT}/\text{Planck})^\sigma \sim \mathcal{O}(10^{-5})^\sigma$, our finding becomes non-trivial. Especially, the results obtained in this work should be generalized directly to other inflationary models with large non-Gaussianities, in which the inverse volume corrections also becomes large therein. In addition, in this paper we only investigated the non-Gaussianities from the inverse volume corrections, while ignored those from the holonomy corrections, in which the sound speed of scalar perturbations are typically changed. Besides, we only investigated in this work the bispectrum from scalar modes, left those from tensor mode unexplored. These topics are worth investigating further.

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Appendix A: Interacting Hamiltonian

In this appendix, we derive the perturbed Hamiltonian density and the perturbed diffeomorphism constraint up to the third order. According to [15], the classical Hamiltonian includes two parts

$$\mathscr{H}[N] = \mathscr{H}_{\rm grav}[N] + \mathscr{H}_{\rm matter}[N] \tag{A1}$$

The gravitational Hamiltonian can be expressed in terms of the extrinsic curvature

$$\mathscr{H}_{\rm grav}[N] = \frac{1}{2}\int_\Sigma d^3x N\mathfrak{H} = \frac{1}{2}\int_\Sigma d^3x N\epsilon_i^{jk}\frac{E^c_jE^d_k}{\sqrt{|\det E|}}\Big[2\partial_c\Gamma^i_d + \epsilon^i_{mn}(\Gamma^m_c\Gamma^n_d - K^m_cK^n_d)\Big], \tag{A2}$$

and the matter part of the Hamiltonian is

$$\mathscr{H}_{\rm matter}[N] = \int_\Sigma d^3xN(\mathfrak{H}_\pi + \mathfrak{H}_\nabla + \mathfrak{H}_\varphi), \tag{A3}$$

where

$$\mathfrak{H}_\pi = \frac{\pi^2}{2\sqrt{|\det E|}},\ \mathfrak{H}_\nabla = \frac{E^a_iE^b_i\partial_a\varphi\partial_b\varphi}{2\sqrt{|\det E|}},\ \mathfrak{H}_\varphi = \sqrt{|\det E|}V(\varphi). \tag{A4}$$

The diffeomorphism constraint is

$$\mathscr{D}_{grav}[N^a] := \int_\Sigma d^3xN^a\Big[(\partial_aA^j_b - \partial_bA^j_a)E^b_j - A^j_a\partial_bE^b_j\Big], \tag{A5}$$

and the matter contribution is

$$\mathscr{D}_{\rm matter}[N^a] := \int_\Sigma d^3xN^a\pi\partial_a\varphi. \tag{A6}$$

In order to obtain the perturbed Hamiltonian density and the perturbed diffeomorphism constraint up to the third order, we need the following two relations. We expand $(\det E)^{\frac{1}{2}}$ and $(\det E)^{-\frac{1}{2}}$ to the third order as follows.

$$\begin{aligned}(\det E)^{\frac{1}{2}} = \bar{p}^{\frac{3}{2}}\Big[\quad & 1 + \frac{1}{2\bar{p}}\delta^i_a\delta E^a_i + \frac{1}{8\bar{p}^2}(\delta^i_a\delta E^a_i)^2 - \frac{1}{4\bar{p}^2}\delta^i_b\delta^j_a\delta E^a_i\delta E^b_j + \frac{1}{48\bar{p}^3}(\delta^k_c\delta E^c_k)^3 \\ & - \frac{1}{8\bar{p}^3}(\delta^k_c\delta E^c_k)(\delta^i_b\delta^j_a\delta E^a_i\delta E^b_j) + \frac{1}{12\bar{p}^3}\delta^i_c\delta^k_b\delta^j_a\delta^k_a\delta E^a_i\delta E^b_j\delta E^c_k \\ & + \frac{1}{12\bar{p}^3}\delta^i_b\delta^j_c\delta E^a_i\delta E^b_j\delta E^c_k + \ldots\Big],\end{aligned} \tag{A7}$$

and

$$(\det E)^{-\frac{1}{2}} = \bar{p}^{-\frac{3}{2}}\Big[\quad 1 - \frac{1}{2\bar{p}}\delta^i_a\delta E^a_i + \frac{1}{8\bar{p}^2}(\delta^i_a\delta E^a_i)^2 + \frac{1}{4\bar{p}^2}\delta^i_b\delta^j_a\delta E^a_i\delta E^b_j - \frac{1}{48\bar{p}^3}(\delta^i_a\delta E^a_i)^3$$

$$- \frac{1}{8\bar{p}^3}(\delta^k_c\delta E^c_k)(\delta^i_b\delta^j_a\delta E^a_i\delta E^b_j) - \frac{1}{12\bar{p}^3}\delta^i_c\delta^k_b\delta^j_a\delta E^a_i\delta E^b_j\delta E^c_k$$

$$- \frac{1}{12\bar{p}^3}\delta^i_b\delta^j_c\delta^k_a\delta E^a_i\delta E^b_j\delta E^c_k + ...\Big]. \tag{A8}$$

Thus the third order gravitational Hamiltonian density can be written as

$$\mathfrak{H}^{(3)}_{\rm grav} = \mathfrak{H}^{(3)}_1 + \mathfrak{H}^{(3)}_2 + \mathfrak{H}^{(3)}_3 \tag{A9}$$

where

$$\begin{aligned}
\mathfrak{H}^{(3)}_1 \;&:=\; \left[\epsilon_i^{\ jk}\frac{E^c_jE^d_k}{\sqrt{\det E}}2\partial_c\Gamma^i_d\right]^{(3)} \\
&=\; 2\epsilon_i^{\ jk}\frac{\delta E^c_j\delta E^d_k}{\sqrt{\det E}}\partial_c\delta\Gamma^i_d + 2\epsilon_i^{\ jk}\delta E^c_j\bar{E}^d_k[(\det E)^{-\frac{1}{2}}]^{(1)}\partial_c\delta\Gamma^i_d \\
&\quad +2\epsilon_i^{\ jk}\bar{E}^c_j\delta E^d_k[(\det E)^{-\frac{1}{2}}]^{(1)}\partial_c\delta\Gamma^i_d + 2\epsilon_i^{\ jk}\bar{E}^c_j\bar{E}^d_k[(\det E)^{-\frac{1}{2}}]^{(2)}\partial_c\delta\Gamma^i_d \\
&=\; -\frac{1}{\bar{p}^{\frac{5}{2}}}\delta^{kl}\delta^j_d\delta E^d_l\delta E^c_j(\partial_c\partial_e\delta E^e_k) + \frac{1}{2\bar{p}^{\frac{5}{2}}}\delta^{kl}\delta E^c_l\delta^m_a\delta E^a_m(\partial_c\partial_e\delta E^e_k) \\
&\quad -\frac{1}{4\bar{p}^{\frac{5}{2}}}\delta^{cl}(\delta^m_a\delta E^a_m)^2(\partial_c\partial_e\delta E^e_l) + \frac{1}{2\bar{p}^{\frac{5}{2}}}\delta^{cl}\delta^m_b\delta E^a_m\delta E^b_n(\partial_c\partial_e\delta E^e_l),
\end{aligned} \tag{A10}$$

$$\begin{aligned}
\mathfrak{H}^{(3)}_2 \;&:=\; \left[\epsilon_i^{\ jk}\frac{E^c_jE^d_k}{\sqrt{\det E}}\epsilon^i_{\ mn}\Gamma^m_c\Gamma^n_d\right]^{(3)} \\
&=\; \epsilon_i^{\ jk}\frac{\bar{E}^d_k\delta E^c_j}{\sqrt{\det E}}\epsilon^i_{\ mn}\delta\Gamma^m_c\delta\Gamma^n_d + \epsilon_i^{\ jk}\frac{\bar{E}^c_j\delta E^d_k}{\sqrt{\det E}}\epsilon^i_{\ mn}\delta\Gamma^m_c\delta\Gamma^n_d \\
&\quad +\epsilon_i^{\ jk}\bar{E}^c_j\bar{E}^d_k[(\det E)^{-\frac{1}{2}}]^{(1)}\epsilon^i_{\ mn}\delta\Gamma^m_c\delta\Gamma^n_d \\
&=\; \frac{1}{4\bar{p}^{\frac{5}{2}}}\delta^k_d\delta E^d_k\delta^{ij}(\partial_c\delta E^c_j)(\partial_a\delta E^a_i) - \frac{1}{2\bar{p}^{\frac{5}{2}}}\delta^i_c\delta^{kj}\delta E^c_j(\partial_b\delta E^b_k)(\partial_a\delta E^a_i),
\end{aligned} \tag{A11}$$

$$
\begin{aligned}
\mathfrak{H}_3^{(3)} &:= \Big[-\epsilon_i^{jk}\frac{E_j^c E_k^d}{\sqrt{\det E}}\epsilon^i_{mn}K_c^m K_d^n\Big]^{(3)} \\
&= -\epsilon_i^{jk}\frac{\delta E_j^c\delta E_k^d}{\sqrt{\det E}}\epsilon^i_{mn}\bar{K}_d^n\delta K_c^m - \epsilon_i^{jk}\frac{\bar{E}_k^d\delta E_j^c}{\sqrt{\det E}}\epsilon^i_{mn}\delta K_c^m\delta K_d^n \\
&\quad -\epsilon_i^{jk}\frac{\delta E_j^c\delta E_k^d}{\sqrt{\det E}}\epsilon^i_{mn}\bar{K}_c^m\delta K_d^n - \epsilon_i^{jk}\frac{\bar{E}_j^c\delta E_k^d}{\sqrt{\det E}}\epsilon^i_{mn}\delta K_c^m\delta K_d^n \\
&\quad -\epsilon_i^{jk}\delta E_j^c\delta E_k^d[(\det E)^{-\frac{1}{2}}]^{(1)}\epsilon^i_{mn}\bar{K}_c^m\bar{K}_d^n - \epsilon_i^{jk}\bar{E}_j^c\bar{E}_k^d[(\det E)^{-\frac{1}{2}}]^{(1)}\epsilon^i_{mn}\delta K_c^m\delta K_d^n \\
&\quad -\epsilon_i^{jk}\delta E_j^c\bar{E}_k^d[(\det E)^{-\frac{1}{2}}]^{(1)}\epsilon^i_{mn}\delta K_c^m\bar{K}_d^n - \epsilon_i^{jk}\delta E_j^c\bar{E}_k^d[(\det E)^{-\frac{1}{2}}]^{(1)}\epsilon^i_{mn}\bar{K}_c^m\delta K_d^n \\
&\quad -\epsilon_i^{jk}\bar{E}_j^c\delta E_k^d[(\det E)^{-\frac{1}{2}}]^{(1)}\epsilon^i_{mn}\bar{K}_d^n\delta K_c^m - \epsilon_i^{jk}\bar{E}_j^c\delta E_k^d[(\det E)^{-\frac{1}{2}}]^{(1)}\epsilon^i_{mn}\bar{K}_c^m\delta K_d^n \\
&\quad -\epsilon_i^{jk}\bar{E}_k^d\delta E_j^c[(\det E)^{-\frac{1}{2}}]^{(2)}\epsilon^i_{mn}\bar{K}_c^m\bar{K}_d^n - \epsilon_i^{jk}\bar{E}_j^c\delta E_k^d[(\det E)^{-\frac{1}{2}}]^{(2)}\epsilon^i_{mn}\bar{K}_c^m\bar{K}_d^n \\
&\quad -\epsilon_i^{jk}\bar{E}_j^c\bar{E}_k^d[(\det E)^{-\frac{1}{2}}]^{(2)}\epsilon^i_{mn}\delta K_c^m\bar{K}_d^n - \epsilon_i^{jk}\bar{E}_j^c\bar{E}_k^d[(\det E)^{-\frac{1}{2}}]^{(2)}\epsilon^i_{mn}\bar{K}_c^m\delta K_d^n \\
&\quad -\epsilon_i^{jk}\bar{E}_j^c\bar{E}_k^d[(\det E)^{-\frac{1}{2}}]^{(3)}\epsilon^i_{mn}\bar{K}_c^m\bar{K}_d^n \\
&= -\frac{2\bar{k}}{\bar{p}^{\frac{3}{2}}}\delta_d^k\delta E_k^d\delta E_j^c\delta K_c^j + \frac{2\bar{k}}{\bar{p}^{\frac{3}{2}}}\delta_d^j\delta E_j^c\delta E_k^d\delta K_c^k - \frac{2}{\bar{p}^{\frac{1}{2}}}\delta_n^d\delta K_d^n\delta E_j^c\delta K_c^j \\
&\quad +\frac{2}{\bar{p}^{\frac{1}{2}}}\delta_m^d\delta E_j^c\delta K_c^m\delta K_d^j + \frac{\bar{k}^2}{2\bar{p}^{\frac{5}{2}}}(\delta_c^j\delta E_j^c)^3 - \frac{\bar{k}^2}{2\bar{p}^{\frac{5}{2}}}\delta_d^j\delta_c^k\delta_a^l\delta E_j^c\delta E_k^d\delta E_l^a \\
&\quad +\frac{1}{2\bar{p}^{\frac{1}{2}}}(\delta_m^c\delta K_c^m)^2\delta_a^l\delta E_l^a - \frac{1}{2\bar{p}^{\frac{1}{2}}}\delta_n^c\delta_m^d\delta K_c^m\delta K_d^n\delta_a^l\delta E_l^a + \frac{2\bar{k}}{\bar{p}^{\frac{3}{2}}}\delta E_j^c\delta K_c^j\delta_a^l\delta E_l^a \\
&\quad +\frac{\bar{k}}{\bar{p}^{\frac{3}{2}}}(\delta_c^j\delta E_j^c)^2(\delta_n^d\delta K_d^n) - \frac{\bar{k}}{\bar{p}^{\frac{3}{2}}}\delta_n^j\delta_c^d\delta E_j^c\delta K_d^n\delta_a^l\delta E_l^a - \frac{1}{2}\frac{\bar{k}^2}{\bar{p}^{\frac{5}{2}}}(\delta_c^j\delta E_j^c)^3 \\
&\quad -\frac{\bar{k}^2}{\bar{p}^{\frac{5}{2}}}(\delta_b^l\delta_a^i\delta E_l^a\delta E_i^b)(\delta_c^j\delta E_j^c) - \frac{\bar{k}}{2\bar{p}^{\frac{3}{2}}}(\delta_a^l\delta E_l^a)^2(\delta_n^d\delta K_d^n) - \frac{\bar{k}}{\bar{p}^{\frac{3}{2}}}(\delta_b^l\delta_a^j\delta E_l^a\delta E_j^b)(\delta_n^d\delta K_d^n) \\
&\quad +\frac{\bar{k}^2}{8\bar{p}^{\frac{5}{2}}}(\delta_a^i\delta E_i^a)^3 + \frac{3\bar{k}^2}{4\bar{p}^{\frac{5}{2}}}(\delta_b^j\delta E_j^b)(\delta_c^i\delta_a^k\delta E_i^a\delta E_k^c) \\
&\quad +\frac{\bar{k}^2}{2\bar{p}^{\frac{5}{2}}}\delta_c^i\delta_b^k\delta_a^j\delta E_i^a\delta E_j^b\delta E_k^c + \frac{\bar{k}^2}{2\bar{p}^{\frac{5}{2}}}\delta_b^i\delta_c^j\delta_a^k\delta E_i^a\delta E_j^b\delta E_k^c \qquad . \qquad \text{(A12)}
\end{aligned}
$$

The matter hamiltonian can be expressed as

$$
\begin{aligned}
\mathfrak{H}_\pi^{(3)} &= \frac{1}{2}\delta\pi^2[(\det E)^{-\frac{1}{2}}]^{(1)} + \bar{\pi}\delta\pi[(\det E)^{-\frac{1}{2}}]^{(2)} + \frac{1}{2}\pi^2[(\det E)^{-\frac{1}{2}}]^{(3)} \\
&= -\frac{1}{4\bar{p}^{\frac{5}{2}}}\delta_a^i\delta\pi^2\delta E_i^a + \frac{\bar{\pi}}{\bar{p}^{\frac{3}{2}}}\delta\pi\Big[\frac{(\delta_a^i\delta E_i^a)^2}{8\bar{p}^2} + \frac{\delta_b^i\delta_a^j\delta E_i^a\delta E_j^b}{4\bar{p}^2}\Big] + \frac{\bar{\pi}^2}{2\bar{p}^{\frac{3}{2}}}\cdot \\
&\Big[-\frac{(\delta_a^i\delta E_i^a)^3}{48\bar{p}^3} - \frac{(\delta_c^i\delta_a^k\delta E_i^a\delta E_k^c)(\delta_b^j\delta E_j^b)}{8\bar{p}^3} - \frac{\delta_c^i\delta_b^k\delta_a^j\delta E_i^a\delta E_j^b\delta E_k^c}{12\bar{p}^3} - \frac{\delta_b^i\delta_c^j\delta_a^k\delta E_i^a\delta E_j^b\delta E_k^c}{12\bar{p}^3}\Big], \quad \text{(A13)}
\end{aligned}
$$

$$\begin{aligned}\mathfrak{H}_{\nabla}^{(3)} &= \frac{\delta^{ij}\bar{E}_j^b\delta E_i^a\partial_a\delta\varphi\partial_b\delta\varphi}{2\sqrt{\det E}} + \frac{\delta^{ij}\bar{E}_i^a\delta E_j^b\partial_a\delta\varphi\partial_b\delta\varphi}{2\sqrt{\det E}} + \frac{\delta^{ij}\bar{E}_i^a\bar{E}_j^b}{2}[(\det E)^{-\frac{1}{2}}]^{(1)}\partial_a\delta\varphi\partial_b\delta\varphi \\ &= \frac{\delta^{ai}\delta E_i^b\partial_a\delta\varphi\partial_b\delta\varphi}{\bar{p}^{\frac{1}{2}}} - \frac{\delta_c^k\delta E_k^c\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi}{4\bar{p}^{\frac{1}{2}}}, \end{aligned} \tag{A14}$$

$$\begin{aligned}\mathfrak{H}_{\varphi}^{(3)} &= \frac{1}{3!}\sqrt{\det E}V_{,\varphi\varphi\varphi}(\bar{\varphi})\delta\varphi^3 + \frac{1}{2}[(\det E)^{\frac{1}{2}}]^{(1)}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2 + [(\det E)^{\frac{1}{2}}]^{(2)}V_{,\varphi}(\bar{\varphi}) \\ &\quad + \left[(\det E)^{\frac{1}{2}}\right]^{(3)}V(\bar{\varphi}) \\ &= \frac{1}{6}\bar{p}^{\frac{3}{2}}V_{,\varphi\varphi\varphi}(\bar{\varphi})\delta\varphi^3 + \frac{1}{4}\bar{p}^{\frac{1}{2}}\delta_a^i\delta E_i^aV_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2 + \left[\frac{(\delta_a^i\delta E_i^a)^2}{8\bar{p}^{\frac{1}{2}}}\right. \\ &\quad \left. -\frac{\delta_b^i\delta_a^j\delta E_i^a\delta E_j^b}{4\bar{p}^{\frac{1}{2}}}\right]V_{,\varphi}(\bar{\varphi})\delta\varphi + \bar{p}^{\frac{3}{2}}\left[\frac{(\delta_c^k\delta E_k^c)^3}{48\bar{p}^3} - \frac{(\delta_c^k\delta E_k^c)(\delta_b^i\delta_a^j\delta E_i^a\delta E_j^b)}{8\bar{p}^3}\right. \\ &\quad \left. +\frac{\delta_c^i\delta_b^k\delta_a^j\delta E_i^a\delta E_j^b\delta E_k^c}{12\bar{p}^3} + \frac{\delta_b^i\delta_c^j\delta_a^k\delta E_i^a\delta E_j^b\delta E_k^c}{12\bar{p}^3}\right]V(\bar{\varphi}). \end{aligned} \tag{A15}$$

Combined with the results obtained by [14], we get the perturbed Hamiltonian as follows.

$$\mathscr{H}_{\text{grav}}^{(3)}[N] = \mathscr{H}_{\text{grav}}^{(3)}[\delta N] + \mathscr{H}_{\text{grav}}^{(3)}[\bar{N}], \tag{A16}$$

where

$$\mathscr{H}_{\text{grav}}^{(3)}[\bar{N}] = \frac{1}{2}\int_{\Sigma} d^3x\bar{N}^{(3)}\mathfrak{H}_{\text{grav}}^{(3)}, \tag{A17}$$

and

$$\mathscr{H}_{\text{grav}}^{(3)}[\delta N] = \frac{1}{2}\int_{\Sigma} d^3x\delta N\mathfrak{H}_{\text{grav}}^{(2)}, \tag{A18}$$

and the matter Hamiltonian reads

$$\mathscr{H}_{\text{matter}}^{(3)}[\bar{N}] = \int_{\Sigma} d^3x\bar{N}\left[\mathfrak{H}_{\pi}^{(3)} + \mathfrak{H}_{\nabla}^{(3)} + \mathfrak{H}_{\varphi}^{(3)}\right], \tag{A19}$$

$$\mathscr{H}_{\text{matter}}^{(3)}[\delta N] = \int_{\Sigma} d^3x\delta N\left[\mathfrak{H}_{\pi}^{(2)} + \mathfrak{H}_{\nabla}^{(2)} + \mathfrak{H}_{\varphi}^{(2)}\right]. \tag{A20}$$

Here we have ignored the high order correction terms caused by the inverse volume, such as $\alpha^{(2)}H^{(2)}$, because in the in-in formulism, these terms do not contribute to the non-Gaussianity.

The diffeomorphism constraint up to the third order is

$$\begin{aligned}\mathscr{D}_{\text{grav}}^{(3)}[\delta N^a] &:= \frac{1}{\gamma}\int_{\Sigma} d^3x\delta N^a\Big[\frac{1}{2\bar{p}}\epsilon_b^{ij}(\partial_a\partial_c\delta E_i^c)\delta E_j^b + \gamma\partial_a\delta K_b^j\delta E_j^b - \frac{1}{2\bar{p}}\epsilon_a^{jk}(\partial_b\partial_c\delta E_k^c)\delta E_j^b \\ &\quad -\ \gamma\partial_b\delta K_a^j\delta E_j^b - \frac{1}{2\bar{p}}\epsilon_a^{jk}\partial_c\delta E_k^c\partial_b\delta E_j^b - \gamma\delta K_a^j\partial_b\delta E_j^b\Big], \end{aligned} \tag{A21}$$

$$\mathscr{D}^{(3)}_{\text{matter}}[\delta N^a] := \int_\Sigma d^3x \delta N^a \delta\pi \partial_a \delta\varphi. \tag{A22}$$

Appendix B: Definitions of shape functions

For all the shapes considered in this paper $f(k) = \text{const.}$, so we have

$$S(k_1, k_2, k_3) = \mathscr{S}(\xi, y) \,. \tag{B1}$$

For the local shape

$$S_{\text{local}} = \frac{k_1^3 + k_2^3 + k_3^3}{3k_1k_2k_3} \,, \tag{B2}$$

for the equilateral shape

$$S_{\text{equil}} = \frac{(k_1 + k_2 - k_3)(k_2 + k_3 - k_1)(k_3 + k_1 - k_2)}{k_1k_2k_3} \,, \tag{B3}$$

for the single shape

$$S_{\text{single}} = \frac{(3\epsilon_0 - 2\eta_0)\sum_i k_i^3 + \epsilon_0 \sum_{i\neq j} k_i k_j^2 + 8\epsilon_0 \sum_{i>j} k_i^2 k_j^2/K}{k_1k_2k_3} \,, \tag{B4}$$

for the $\mathcal{F}_1$ shape

$$S_{\mathcal{F}_1} = \left[\left(\frac{k_0}{k_1}\right)^\sigma + \left(\frac{k_0}{k_2}\right)^\sigma + \left(\frac{k_0}{k_3}\right)^\sigma\right] \times S_{\text{single}} \,, \tag{B5}$$

and for the $\mathcal{F}_2$ shape

$$S_{\mathcal{F}_2} = \frac{\left[2\omega_3'(\epsilon_0 - \eta_0) + \omega_2'\epsilon_0\right] \sum_i k_i^3 \left(k_0/k_i\right)^\sigma + \omega_2'\epsilon_0 \sum_{i\neq j} k_j^2 k_i \left(k_0/k_i\right)^\sigma}{k_1k_2k_3} \,. \tag{B6}$$

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